Examples 4

$V_R = 3e^{-2t} u_0(t)$

$R = 1\,\Omega$

energy $0 < t < \infty$.

$P_R = V_R^2 / R = 9e^{-4t} u_0(t)$.

$$W_R = \int_0^\infty V_R^2\, dt \int_0^\infty 9e^{-4t}\, dt = -\frac{9}{4} e^{-4t}\Big|_0^\infty$$

$$= \frac{9}{4} e^{-4t}\Big|_\infty^0 = \frac{9}{4}\left[1 - 0\right]$$

$$= \frac{9}{4} = \underline{\underline{2.25}} \text{ Joules}$$

Parseval's theorem

$$\int_{-\infty}^\infty |f(t)|^2\, dt = \frac{1}{2\pi} \int_{-\infty}^\infty |F(\omega)|^2\, d\omega$$

$$F(\omega) = \mathcal{F}\{3e^{-2t} u_0(t)\} = \frac{3}{j\omega + 2}$$

$$|F(\omega)|^2 = \underline{F(\omega)}\,\underline{F^*(\omega)} = \frac{9}{\omega^2 + 2^2}$$

$$W_R = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{9}{\omega^2+2^2}\, d\omega.$$

$$= \frac{2}{2\pi}\int_{0}^{\infty} \frac{9}{\omega^2+2^2}\, d\omega = \frac{9}{\pi}\int_{0}^{\infty} \frac{1}{\omega^2+2^2}\, d\omega$$

$$\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a} \arctan \frac{x}{a} + C$$

$$\frac{9}{\pi}\left[\frac{1}{2} \arctan \frac{\omega}{2}\right]_0^{\infty} = \frac{9}{2\pi}\cdot\frac{\pi}{2} = 2.25 \text{ joules.}$$

$$\frac{9}{\pi}\left[\frac{1}{2} \arctan \frac{\omega}{2}\right]_0^{\infty} = \frac{9}{\cancel{2\pi}}\,\frac{\cancel{\pi}}{2} = \frac{9}{4} = 2.25$$