Example 3

$$\frac{d^n}{dt^n} f(t) \Leftrightarrow (j\omega)^n F(\omega)$$

$$\frac{d}{dt} v_{out} + 4 v_{out} = 10 v_{in}$$

$$j\omega V_{out}(\omega) + 4 V_{out}(\omega) = 10 V_{in}(\omega)$$

$$(j\omega + 4) V_{out}(\omega) = 10 V_{in}(\omega)$$

$$H(s) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{10}{j\omega + 4}$$

$$V_{in}(\omega) = \mathcal{F}\{v_{in}(t)\} = \mathcal{F}\{3e^{-2t} u_0(t)\} = \frac{3}{j\omega + 2}$$

$$V_{out}(\omega) = H(\omega) \cdot V_{in}(\omega) = \left(\frac{10}{j\omega + 4}\right)\left(\frac{3}{j\omega + 2}\right)$$

$$= \frac{r_1}{j\omega + 4} + \frac{3}{j\omega + 2}$$

$$r_1 = -15 \qquad r_2 = 15$$

$$V_{out}(\omega) = \frac{15}{j\omega + 2} - \frac{15}{j\omega + 4}$$

$$V_{out}(t) = \mathcal{F}^{-1}\left\{ \frac{15}{jw+2} - \frac{15}{jw+4} \right\}$$

$$= 15\left(e^{-2t} - e^{-4t}\right)u_0(t).$$