EG-247

Filters.

Example 1:

find the roots of $P(s)$

$$P(s) = s^2 + \omega_c\sqrt{2}\, s + \omega_c^2 = 0$$

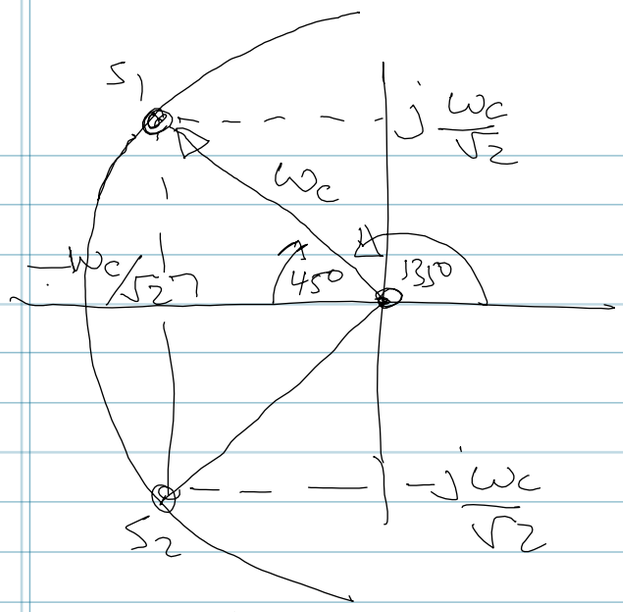
$$s_{1,2} = \frac{-\omega_c\sqrt{2} \pm \sqrt{\omega_c^2 \cdot 2 - 4\omega_c^2}}{2}$$

$$= -\frac{\omega_c}{\sqrt{2}} \pm j\frac{\sqrt{2\omega_c^2}}{2}$$

$$= -\frac{\omega_c}{\sqrt{2}} \pm j\frac{\sqrt{2}\,\omega_c}{2\sqrt{2}}$$

$$s_{1,1} = \omega_c\left(-\frac{1}{\sqrt{2}} \pm j\frac{1}{\sqrt{2}}\right)$$

$$\left(s + \omega_c\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)\right)\left(s + \omega_c\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)\right)$$

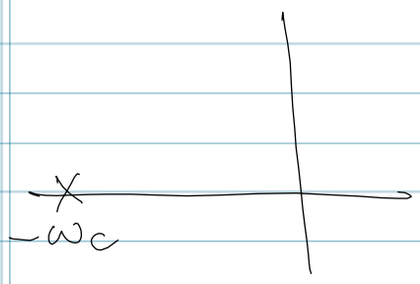


$$135^\circ = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

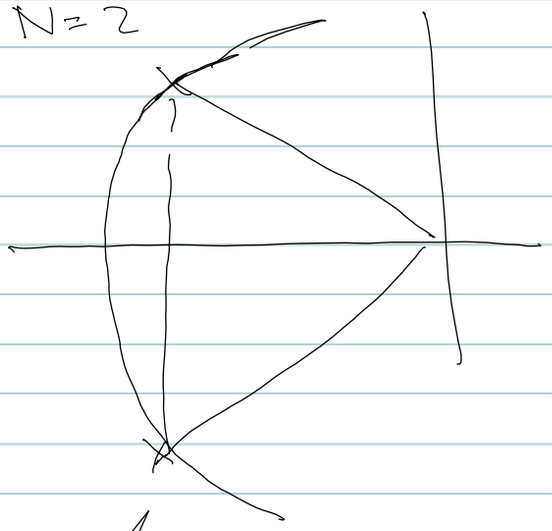
$$s_1 = s_2^*$$

$$\frac{(s - \omega_c e^{j3\pi/4})(s - \omega_c e^{-j3\pi/4})}{}$$

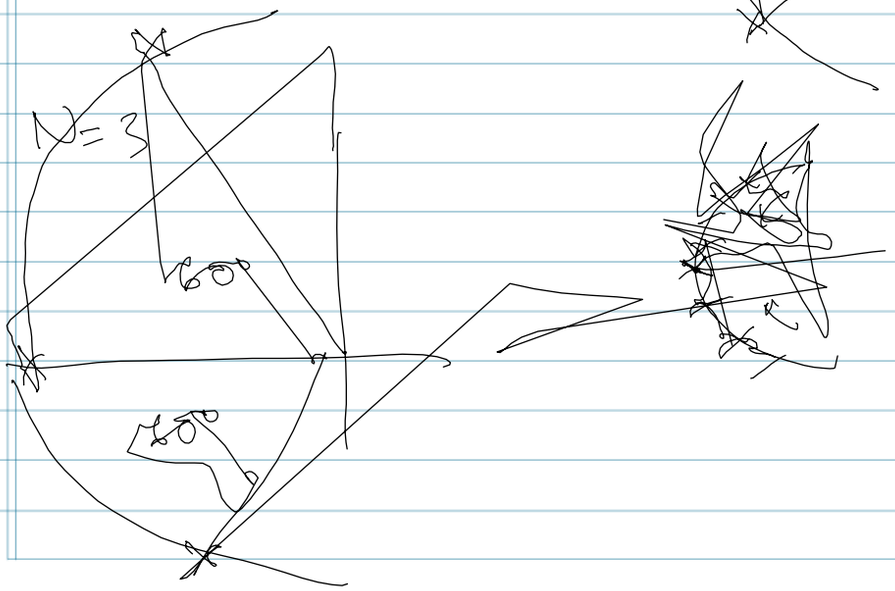
$N=1$



$N=2$



$N=2$



Ex. What is $N=3$?

Example 2

$$(s^2 + \omega_c\sqrt{2}\,s + \omega_c^2)Y(s) = \omega_c^2 U(s)$$

$$\frac{d^2y(t)}{dt^2} + \omega_c\sqrt{2}\frac{dy}{dt} + \omega_c y = \omega_c^2 u(t)$$

$$H_B(s) = \frac{Y(s)}{X(s)} = \frac{\omega_c^2}{s^2 + \omega_c\sqrt{2}s + \omega_c^2}$$

Example 3

$$H_B(j\omega) = \frac{Y(\omega}{X(\omega)} =$$

$$s \Rightarrow j\omega$$

$$H_B(j\omega) = \frac{\omega_c^2}{(j\omega)^2 + \omega_c\sqrt{2}\,j\omega + \omega_c^2}$$

$$H_B\left(\frac{\omega}{\omega_c}\right) = \frac{1}{\left(j\frac{\omega}{\omega_c}\right)^2 + j\sqrt{2}\frac{\omega}{\omega_c} + 1}$$

$$= \frac{1}{-(\omega/\omega_c)^2 + j\sqrt{2}\frac{\omega}{\omega_c} + 1}$$

$$H(\omega) \quad \frac{1}{1-\left(\frac{\omega}{\omega_c}\right)^2+j\sqrt{2}\left(\frac{\omega}{\omega_c}\right)}$$

Example 4

$H(\omega) \rightarrow h(t)$ Inverse fourier

$$e^{-at}\sin \omega_0 u_0(t) \Longleftrightarrow \frac{\omega_0}{(j\omega+a)^2+\omega_0^2}$$

ω_c^2

~~$\omega_c\sqrt{2}$~~

$$(j\omega)^2 + \sqrt{2}\omega_c j\omega + \omega_c^2$$

$$h_B(t) = \sqrt{2}\omega_c e^{\frac{-\omega_c}{\sqrt{2}}t} \sin\left(\frac{\omega_c}{\sqrt{2}}t\right) u_0(t)$$