

$$H(z) = \frac{Y(z)}{U(z)} = \frac{z^2 + z}{z^2 - 0.5z + 0.125}$$

impulse response

$$\mathcal{Z}^{-1}\{H(z)\} = \mathcal{Z}^{-1}\left(\frac{z^2 + z}{z^2 - 0.5z + 0.125}\right)$$

$$\frac{H(z)}{z} = \frac{z + 1}{z^2 + 0.5z + 0.125}$$

(residue)

$$r_1 = 0.5 - j2.5$$

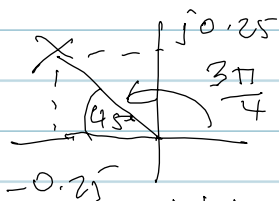
$$r_2 = 0.5 + j2.5$$

$$p_1 = 0.25 + j0.25$$

$$p_2 = 0.25 - j0.25$$

$$\frac{H(z)}{z} = \frac{0.5 + j2.5}{z - 0.25 - j0.25} + \frac{0.5 - j2.5}{z - 0.25 + j0.25}$$

$$H(z) = \frac{(0.5 + j2.5)z}{z - (0.25 + j0.25)} + \frac{(0.5 - j2.5)z}{z - (0.25 - j0.25)}$$

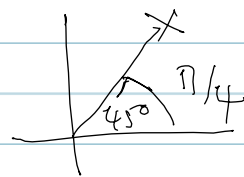


$$\sqrt{0.25^2 + 0.25^2} = 0.25\sqrt{2}$$

$$H(z) = \frac{(0.5 + j2.5)z}{z - 0.25\sqrt{2}e^{j3\pi/4}} + \frac{(0.5 - j2.5)z}{z - 0.25\sqrt{2}e^{-j3\pi/4}}$$

$$\underline{a^n u_0[n]} \Leftrightarrow \frac{z}{z-a}$$

$$h[n] = (0.5 - j2.5)(0.25\sqrt{2} e^{j\frac{\pi}{4}})^n + (0.5 + j2.5)(0.25\sqrt{2} e^{-j\frac{\pi}{4}})^n$$



$$= 0.5(0.25\sqrt{2} e^{j\frac{\pi}{4}})^n - j2.5(0.25\sqrt{2} e^{j\frac{\pi}{4}})^n + 0.5(0.25\sqrt{2} e^{-j\frac{\pi}{4}})^n - j2.5(0.25\sqrt{2} e^{-j\frac{\pi}{4}})^n$$
$$= (0.5)(0.25\sqrt{2})^n \left(e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n} \right)$$

$$+ j2.5(0.25\sqrt{2})^n \left(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n} \right)$$

$$\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos \theta \quad \frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin \theta$$

$$h[n] = \left(\frac{\sqrt{2}}{4} \right)^n \left(\cos n\frac{\pi}{4} + 5 \sin n\frac{\pi}{4} \right)$$

