

### Example 3

$$F(z) = \frac{z+1}{(z-1)(z^2+2z+2)}$$

$$\frac{F(z)}{z} = \frac{z+1}{z(z-1)(z^2+2z+2)}$$

$\underbrace{\hspace{10em}}_{\rightarrow (-1 \pm j)}$

$$= \frac{r_1}{z} + \frac{r_2}{z-1} + \frac{r_3}{z+1-j} + \frac{r_4}{z+1+j}$$

$$r_1 = -0.5$$

$$r_2 = 0.4$$

$$r_3 = \frac{z+1}{(z)(z-1)(z+1+j)} \Big|_{z=-1+j}$$

$$= \frac{j}{(1+j)(-2+j)(j^2)} = \underline{\underline{0.05 + j0.15}}$$

$$r_3 = r_4^* = 0.05 - j0.15$$

$$\frac{F(z)}{z} = \frac{-0.5}{z} + \frac{0.4}{z-1} + \frac{0.05 + j0.15}{(z+1-j)} + \frac{0.05 - j0.15}{z+1+j}$$

$$\hat{F}(z) = -0.5 + \frac{0.4z}{z-1} + \frac{(0.05 + j0.15)z}{z - (-1+j)} + \frac{(0.05 - j0.15)z}{z - (-1-j)}$$

$$z^{-0.5} + \frac{0.4z}{z-1} + \frac{(0.05 + j0.15)z}{z - \sqrt{2}e^{j\frac{3\pi}{4}}} + \frac{(0.05 - j0.15)}{z - \sqrt{2}e^{-j\frac{3\pi}{4}}}$$

$$\delta[n] \Leftrightarrow 1$$

$$a^n u_0[n] \Leftrightarrow \frac{z}{z-a}$$

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$$f[n] = -0.5\delta[n] + 0.4(1)^n + (0.05 + j0.15)(\sqrt{2}e^{j\frac{3\pi}{4}})^n + (0.05 - j0.15)(\sqrt{2}e^{-j\frac{3\pi}{4}})^n$$

$$\dots + (0.05 - j0.15)(\sqrt{2}e^{+j\frac{3\pi}{4}})^n$$

$$+ (0.05 - j0.15)(\sqrt{2}e^{-j\frac{3\pi}{4}})^n$$

$$0.05\sqrt{2}^n \quad \dots \quad j0.15\sqrt{2}^n \quad \dots$$

$$f[n] = -0.5\delta[n] + 0.4 + \frac{\sqrt{2}^n}{10} \cos n\frac{3\pi}{4} - \frac{3\sqrt{2}^n}{10} \sin n\frac{3\pi}{4}$$

$$\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos \theta$$

$$\frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin \theta$$